Lebanese University
Faculty of information
Section 1

Final Exam: Discrete Mathematics **Time: 120 minutes**

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Problem 1.

1. Determine whether each of these compound propositions are tautologies.
   a) $(p \wedge q) \rightarrow (p \vee q)$
   b) $[(p \vee q) \rightarrow r] \rightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$

2. Use the logical equivalences above to show that $\neg(p \vee \neg(p \wedge q))$ is a contradiction.

3. Perform the indicated bit string operations. The bit strings are given in groups of four bits each for ease of reading.
   (10100111 ⊕ 11100010) ∧ 11111000 0 1 000 000
   (10101010 ∧ 01110010) ∨ 11011001 11111 011

4. Express the negation of the statement $\forall x((x \geq 4) \wedge (x \leq 7))$ in terms of quantifiers without using the negation symbol. $\exists x\ (x < 4 \vee x > 7)$

Problem 2.

1. Let a and b are integers. Show that $ab = \gcd(a, b) . lcm(a, b)$.
2. Convert the octal expansion of the integer to a binary expansion
   $(5646)_8$ $(2982)_{10}$ $(101110100110)_2$

3.
   a) Find an inverse of a modulo m for each of these pairs of relatively prime integers.
   $a = 124, m = 203.$ -18
   b) Solve each of these congruences using the modular inverses found in part (a)
   $124x \equiv 3 \pmod{203}$ $x \equiv 149 \pmod{203}$

Problem 3.

1. Let R_1 and R_2 be the "congruent modulo 3" and "the congruent modulo 8" relations, respectively, on the set of integers. That is,

$$R_1 = \{(a, b)/a \equiv b \pmod 4\}$$
$$R_2 = \{(a, b)/a \equiv b \pmod 6\}$$

Find

a) $R_1 \cup R_2$

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b) $R_1 \cap R_2$
c) $R_1 - R_2$
d) $R_1 \oplus R_2$

2. Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Find the matrix representing

a) $R_1 \cup R_2$ and $R_1 \cap R_2$
b) $R_1 \circ R_2$
c) $R_1 \oplus R_2$
d) $\overline{R_2}$ and R_1^{-1}

Good Luck